Available online at www.ijpab.com

DOI: http://dx.doi.org/10.18782/2320-7051.6842

ISSN: 2320 – 7051

Int. J. Pure App. Biosci. **6 (5):** 135-138 (2018)

Research Article

International Journal of Pure & Applied Bioscience

OPEN ACCESS

An Improvement in Variance Estimator for the Estimation of Population Variance, Using Known Values of Auxiliary Information

M. A. Bhat*, S. Maqbool and Mir Subzar

Division of Agricultural Economics and Statistics Faculty of Agriculture Wadura Skuast-Kashmir

*Corresponding Author E-mail: mabhat.1500@gmail.com

Received: 30.08.2018 | Revised: 27.09.2018 | Accepted: 6.10.2018

ABSTRACT

In this research paper, we have developed an efficient estimator by introducing the linear combination of deciles arithmetic mean and coefficient of kurtosis as auxiliary information to achieve efficiency of the estimator for the estimation of population variance. The bias and mean square error has been derived up to the first order of approximation. The empirical study has been carried out through numerical demonstration to justify the performance of new developed estimators.

Key words: *Deciles arithmetic mean, Coefficient of kurtosis, Bias, Mean Square Error, Efficiency.*

INTRODUCTION

In survey sampling, the functions of population parameters of auxiliary variable have great role in modifying and developing the new estimators when there exists a close association between auxiliary and study variables. Different authors have utilized this auxiliary information in different ways in order to increase the efficiency of estimators. Some of them from literature who have addressed the issue of utilizing supplementary information are Isaki[1] who has utilized standard deviation as auxiliary information to enhance the efficiency of the estimator for the estimation of population variance. Upadhyaya and Singh[2] have incorporated the coefficient of kurtosis as auxiliary variable to increase the efficiency of estimator for the estimation of population variance. Kadilar and Cingi[3] introduced the coefficient of variation as auxiliary variable to improve the efficiency of the estimator. On the same lines, authors such as Sarandal CE[4], M.A. Bhat[5] have also utilized this auxiliary information to enhance the efficiency of estimators The Strategy of modifying estimators, by using auxiliary information is now being regularly carried out in the field of survey sampling to improve the efficiency of estimators in order to have the precise and reliable estimates in survey estimation.

Let us consider a finite population having N distinct and identifiable units. Let $Y_i(i=1,2,3,,,,,,,N)$ denote the observations on study variable Y and $X_i(i=1,2,3....N)$ denote the observations on auxiliary variable (X), when the information about auxiliary variable is known.

Cite this article: Bhat, M. A., Maqbool, S. and Subzar, M., An Improvement in Variance Estimator for the Estimation of Population Variance, Using Known Values of Auxiliary Information, *Int. J. Pure App. Biosci.* **6(5):** 135-138 (2018). doi: http://dx.doi.org/10.18782/2320-7051.6842

In this Study our aim is to estimate the finite population variance by introducing new and improved estimators in sample survey's.

MATERIAL AND METHODS

Notations

$N = population\ \ size,\ n = Sample\ \ size,\ \gamma = \dfrac{1}{n}, Y = study\ \ variable, X = Auxiliary\ \ variable,$

$\bar{X}\ \&\ \bar{Y} = Population\ \ means, \bar{x}\ \&\ \bar{y} = Sample\ \ means, S_y^2\ \&\ S_x^2 = Population\ \ variances,$

$C_x\ \&\ C_y = Cofficient\ \ of\ \ variations, \rho = correlation\ \ cofficient,\ \ \beta_{1x} = Sekewness\ \&$

$\beta_{2x} = Kurtosis$

$D = Decile, D_{A.M} = Decile\ \ airthmetic\ \ mean,\ B(.) = Bias\ \&\ MSE(.) = Mean\ \ square\ \ error$

Proposed Estimator:- $\hat{S}_M^2 = s_y^2\left[\dfrac{S_x^2 + (D_{A.M}.\beta_{2x})}{s_x^2 + (D_{A.M}.\beta_{2x})}\right]$

We have derived the bias and mean square error of the proposed estimator up to the first order of approximation as given below:

Let $e_0 = \dfrac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \dfrac{s_x^2 - S_x^2}{S_x^2}$. Further we can write $s_y^2 = S_y^2(1+e_0)$ and $s_x^2 = S_x^2(1+e_1)$ and from the definition of e_0 and e_1 we obtain:

$E[e_0] = E[e_1] = 0$, $\quad E[e_0^2] = \dfrac{1-f}{n}(\beta_{2(y)} - 1)$, $\quad E[e_1^2] = \dfrac{1-f}{n}(\beta_{2(x)} - 1)$,

$E[e_0 e_1] = \dfrac{1-f}{n}(\lambda_{22} - 1)$

The proposed estimator is given as:

$$\hat{S}_M^2 = s_y^2\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + \alpha a_i}\right] \tag{4.1}$$

$$\Rightarrow \quad \hat{S}_M^2 = s_y^2(1+e_0)\left[\frac{S_x^2 + \alpha a_i}{s_x^2 + e_1 S_x^2 + \alpha a_i}\right]$$

$$\Rightarrow \quad \hat{S}_M^2 = \frac{S_y^2(1+e_0)}{(1+A_M e_1)} \text{ Where } A_M = \frac{S_x^2}{S_x^2 + \alpha a_i}$$

$$a_i = (D_{A.M}.\beta_{2x}) and \alpha = 1$$

$$\Rightarrow \quad \hat{S}_M^2 = S_y^2(1+e_0)(1+A_M e_1)^{-1} \tag{4.2}$$

$$\Rightarrow \quad \hat{S}_M^2 = S_y^2(1+e_0)(1 - A_M e_1 + A_M^2 e_1^2 - A_M^3 e_1^3 + ...) \tag{4.3.}$$

Expanding and neglecting the terms more than 3rd order, we get

$$\hat{S}_M^2 = S_y^2 + S_y^2 e_0 - S_y^2 A_M e_1 - S_y^2 A_M e_0 e_1 + S_y^2 A_M^2 e_1^2 \tag{4.5}$$

$$\Rightarrow \quad \hat{S}_M^2 - S_y^2 = S_y^2 e_0 - S_y^2 A_M e_1 - S_y^2 A_M e_0 e_1 + S_y^2 A_M^2 e_1^2 \tag{4.6}$$

By taking expectation on both sides of (4.6), we get

$$E(\hat{S}_M^2 - S_y^2) = S_y^2 E(e_0) - S_y^2 A_M E(e_1) - S_y^2 A_M E(e_0 e_1) + S_y^2 A_M^2 E(e_1^2) \tag{4.7}$$

$$Bias(\hat{S}_M^2) = S_y^2 A_M^2 E(e_1^2) - S_y^2 A_M E(e_0 e_1) \tag{4.8}$$

$$Bias\left(\hat{S}_M^2\right) = \gamma S_y^2 A_M \left[A_M\left(\beta_{2x}-1\right)-\left(\lambda_{22}-1\right)\right] \tag{4.9}$$

Squaring both sides of (4.7) and neglecting the terms more than 2nd order and taking expectation, we get

$$E\left(\hat{S}_M^2 - S_y\right)^2 = \gamma S_y^4 E\left(e_0^2\right) + S_y^4 A_M^2 E(e_1^2) - 2S_y^4 A_M E\left(e_0 e_1\right)$$

$$MSE\left(\hat{S}_M^2\right) = \gamma S_y^4\left[(\beta_{2y}-1) + A_M^2\left(\beta_{2x}-1\right) - 2A_M\left(\lambda_{22}-1\right)\right]$$

Efficiency conditions: - we have derived the efficiency conditions of proposed estimators with other existing estimators under which proposed estimators are performing better than the existing estimators

The bias and Mean square error of existing ratio type estimators up to the first order of approximation is given by

$$Bias\left(\hat{S}_K^2\right) = \gamma S_y^2 R_K\left[R_K\left(\beta_{2x}-1\right)-\left(\lambda_{22}-1\right)\right] \tag{5.1}$$

$$MSE\left(\hat{S}_K^2\right) = \gamma S_y^4\left[\left(\beta_{2y}-1\right) + R_K^2\left(\beta_{2x}-1\right) - 2R_K\left(\lambda_{22}-1\right)\right] \tag{5.2}$$

$R_K = Existing.cons\tan t$

$, K = 1,2,3,4........$

Bias, MSE and constant of proposed estimators is given by

$$Bias\left(\hat{S}_P^2\right) = \gamma S_y^2 R_P\left[R_P\left(\beta_{2x}-1\right)-\left(\lambda_{22}-1\right)\right] \tag{5.3}$$

$$MSE\left(\hat{S}_P^2\right) = \gamma S_y^4\left[\left(\beta_{2y}-1\right) + R_P^2\left(\beta_{2x}-1\right) - 2R_P\left(\lambda_{22}-1\right)\right] \tag{5.4}$$

$R_P = proposed.cons\tan t$

$P = 1,2,3............$

From Equation (2) and (3), we have

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right) if \lambda_{22} \ge 1 + \frac{\left(R_P + R_K\right)\left(\beta_{2x}-1\right)}{2}$$

$$MSE\left(\hat{S}_P^2\right) \le MSE\left(\hat{S}_K^2\right)$$

$$\gamma S_y^4\left[\left(\beta_{2y}-1\right) + R_P^2\left(\beta_{2x}-1\right) - 2R_P\left(\lambda_{22}-1\right)\right] \le \gamma S_y^4\left[\left(\beta_{2y}-1\right) + R_K^2\left(\beta_{2x}-1\right) - 2R_K\left(\lambda_{22}-1\right)\right] \tag{5.5}$$

$$\Rightarrow \left[\left(\beta_{2y}-1\right) + R_P^2\left(\beta_{2x}-1\right) - 2R_P\left(\lambda_{22}-1\right)\right] \le \left[\left(\beta_{2y}-1\right) + R_K^2\left(\beta_{2x}-1\right) - 2R_K\left(\lambda_{22}-1\right)\right] \tag{5.6}$$

$$\Rightarrow \left[1 + R_P^2\left(\beta_{2x}-1\right) - 2R_P\left(\lambda_{22}-1\right)\right] \le \left[1 + R_K^2\left(\beta_{2x}-1\right) - 2R_K\left(\lambda_{22}-1\right)\right] \tag{5.7}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2 - R_K^2\right)\left[-2R_P\left(\lambda_{22}-1\right)\right] \le \left[-2R_K\left(\lambda_{22}-1\right)\right] \tag{5.8}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2 - R_K^2\right)\left[-2\left(\lambda_{22}-1\right)\left(R_P - R_K\right)\right] \le 0 \tag{5.9}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P^2 - R_K^2\right) \le \left[2\left(\lambda_{22}-1\right)\left(R_P - R_K\right)\right] \tag{5.10}$$

$$\Rightarrow \left(\beta_{2x}-1\right) \le \frac{2\left(\lambda_{22}-1\right)\left(R_P - R_K\right)}{\left(R_P^2 - R_K^2\right)} \tag{5.11}$$

$$\Rightarrow \left(\beta_{2x}-1\right) \le \frac{2\left(\lambda_{22}-1\right)\left(R_P - R_K\right)}{\left(R_P - R_K\right)\left(R_P + R_K\right)} \tag{5.12}$$

$$\Rightarrow \left(\beta_{2x}-1\right)\left(R_P + R_K\right) \le 2\left(\lambda_{22}-1\right) \tag{5.13}$$

By solving equation (5.13), we get

$$MSE\left(\hat{S}_P^2\right) \leq MSE\left(\hat{S}_K^2\right) if \lambda_{22} \geq 1 + \frac{(R_P + R_K)(\beta_{2x} - 1)}{2}$$

Numerical illustration:-

$N = 234, n = 35, \overline{Y} = 29.3626, \overline{X} = 245.088, S_y = 51.556, S_x = 596.332, \rho = 0.96, \beta_{2y} = 89.231, \beta_{2x} = 89.189$
$\lambda_{22} = 4.041, \beta_{1x} = 8.83, \beta_{1y} = 8.27, TM = 167.4, Q_1 = 67.75, Q_2 = 113.5, Q_3 = 230.25, C_x = 2.43, D_1 = 49.0,$
$D_2 = 63.0, D_3 = 75.0, D_4 = 90.0, D_5 = 113.5, D_6 = 145.9, D_7 = 197.9, D_8 = 271.1, D_9 = 467.5, D_{10} = 6720.0$

Table-1 Bias and MSE of Existing and proposed estimators

Estimators	Bias	MSE
Existing Estimators		
Isaki[1]	**5494.93**	**29216819.02**
Upadhyaya &Singh[2]	**5483.00**	**29187658.80**
Kadilar & Cingi[3]	**5483.00**	**29187658.80**
Proposed estimator M[4]	**544.27**	**16410339.19**

Table-2 Percent relative efficiency of proposed estimators with existing estimators

$Existing - estimators \rightarrow$	**Isaki**	**Upadhyaya & Singh**	**Kadilar & Cingi**
$\Pr oposed - estimator$	**178.30**	**177.87**	**177.87**

CONCLUSION

Above study clearly reveals that the proposed estimator has shown better performance in terms of bias, mean square error and percent relative efficiency as compared to other existing estimators, which can be easily seen from table-1 and table-2.

REFERENCES

1. Isaki, C. T., Variance estimation using auxiliary information. *Journal of the American Statistical Association*, **78:** 117-123 (1983).
2. Upadhyaya, L. N. and Singh, H. P., use of auxiliary variable in the estimation of population variance, *Mathematical forum,* **4:** 33-36 (1936).
3. Kadilar, C., & Cingi, HImprovement in Variance estimation using auxiliary information, *Hacettepe Journal of mathematics and Statistics*, **35(1):** 117-115 (2006a).
4. Sarandal, C. E., Swenson, B., Wretman Model Assisted Survey Sampling, New York: Springer, (1992).
5. Bhat, M. A., Maqbool, S., Saraf, S. A., Rouf, A. B. and Malik, S. H., *Journal of Advances in Research* **13(2):** 1-6, Article no.37321. ISSN: 2348-0394, NLMID: 101666096 (2018).